Turbo Code Carrier Synchronization Losses (Radio Losses)

by

Shervin Shambayati[1], Peter Kinman[2] and Layla Tadjpour[1]

[1]Jet Propulsion Laboratory

and

[2]Case Western Reserve University

Abstract

In this paper the radio loss results for (8920,1/3), (8920,1/6), (1784,1/3) and (1784,1/6) codes are presented. These radio losses were calculated through simulations for a range of data rates. These simulations included both suppressed carrier modulation and residual carrier modulation cases. The radio losses were calculated for a frame error rate of 3×10^{-4} for (8920,1/3) and (8920,1/6) codes and a frame error rate of 6×10^{-5} for (1784,1/3) and (1784,1/6) codes. The simulations for the residual carrier case were run for loop signal to noise ratios of 13dB, 15dB and 17dB with a loop bandwidth of 10Hz. The simulations for the suppressed carrier case were run for a loop of signal to noise ratio of 17dB. The results of these simulations indicate that the radio losses for turbo codes are low enough to warrant their use in deep space links (maximum of 1dB loss at 17dB loop signal to noise ratio for residual carrier and 1.3dB loss at 17dB loop signal to noise ratio for suppressed carrier at high data rates). Furthermore, these results indicate that by normalizing the radio losses for frame size, loop bandwidth and the loop signal to noise ratio, a single curve could be used for calculating the radio loss for any given data rate at any given loop signal to noise ratio.

I. Introduction

Turbo codes offer an error rate performance near the Shannon limit while having a relatively simple decoding process [1]. Therefore, they are attractive for use in the deep space applications. Questions have arisen about the performance of these codes in such applications due to the unknown effects of receiver tracking phase errors that manifest themselves in the form of radio losses [2]. In this paper we present radio loss results of simulations we have performed for (8920,1/3), (8920,1/6), (1784,1/3) and (1784,1/6). These results indicate that turbo codes have reasonable radio losses over all data rates for reasonable loop bandwidth settings and tracking loop signal to noise ratios (LSNR). Furthermore, they indicate that a simple model could be devised to calculate radio losses for any given frame error rate at any given bandwidth and at any given data rate. In Section II we discuss the simulation approach. In Section III baseline performance of the codes under consideration are presented. In Sections IV and V results for the residual carrier and suppressed carrier simulations are presented, respectively. In Section VI conclusions are drawn.

II. Simulation Approach

For each code two types of simulations were performed: one for the suppressed carrier modulation and another for the residual carrier modulation. For the suppressed carrier case, the phase errors were simulated for a digital Costas loop with 17dB loop signal to noise ratio (LSNR). For the residual carrier case, the phase errors were simulated for a digital phase locked loop with loop signal to noise ratios of 13dB, 15dB and 17dB. For both the suppressed carrier and residual carrier cases the loop bandwidth was set at 10Hz. Furthermore, since the simulations were geared towards deep space applications and Deep Space Network's Block V Receivers, an update rate of 2000Hz for the tracking loops was selected [3].

These simulations were performed to evaluate the radio losses for a given frame error rate (FER). The reason that we use FER instead of bit error rate (BER) as the measure of performance is

---

The work described was funded by the TMOD Technology Program and performed at the Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration.

that the decoder decodes frames and not individual bits. Therefore, if it is discovered that a frame is in error all bits in that frame are suspect and thus, they cannot be used.

For calculation of radio losses we used an FER of 3×10^{-4} for (8920,1/3) and (8920, 1/6) codes and an FER of 6×10^{-5} for (1784,1/3) and (1784,1/6) codes. These values were selected for two reasons: 1. They are low enough to be practical but high enough so that simulation time for collection of statistically significant number of errors would not become prohibitive. 2. 1784-bit block length codes have one fifth as many bits as 8920-bit block length codes, therefore, on the average it takes five times as many 1784-bit frames to send a message as it does sending 8920-bit frames. Assuming independent frame errors, this means that for a message to have the same probability of being received correctly under both schemes, the error rate for the 1784-bit codes should be approximately a fifth of that for 8920-bit codes.

The data rates for which these simulations were performed were varied so that the whole range of radio losses, from low rate to medium rate to high rate, could be calculated.

III. Baseline FER Performance and Some Theoretical Results

Before presenting the simulation results we needed to characterize the performance of each code in the presence of additive white Gaussian noise (AWGN). This was done through simulations. The results of these simulations are presented in Figure 1.

As we can see from Figure 1, in the region of interest for our radio loss simulations, the FER curves are linear in log, i.e., the curves could be accurately described by the function

$$f_{FER}\left(\frac{E_b}{N_0}\right)=\exp\left(\alpha_0-\alpha_1\cdot\frac{E_b}{N_0}\right) \tag{1}$$

in this region. In Equation (1) above $\frac{E_b}{N_0}$ is the bit signal to noise ratio and is _not in dBs_.

Equation (1) was fitted to the FER curves for each code and α_0 and α_1 for each code was calculated. The results of these calculations are presented in Table 1.

Code	α_0	α_1
(8920,1/3)	145.3007	138.9979
(8920,1/6)	129.2476	140.7286
(1784,1/3)	61.82092	58.99851
(1784,1/6)	61.72809	66.21811

Table 1. FER Curve Fit Parameters for Different Turbo Codes

Equation (1) could be modified to calculate the high rate radio loss for each code. The modification consists of setting FER value to 1 for all values of $\frac{E_b}{N_0}$ for which equation (1) is greater than 1, i.e.:

$$f_{FER}\left(\frac{E_b}{N_0}\right)=\begin{cases}1 & \alpha_0/\alpha_1 \geq \frac{E_b}{N_0}\\ \exp\left(\alpha_0-\alpha_1\cdot\frac{E_b}{N_0}\right) & otherwise\end{cases} \quad (2)$$

Equation (2) is then used to calculate the actual FER in presence of phase tracking error for high data rates. Actual methodology for calculation of radio losses has been treated before [2] and is beyond the scope of this paper. It suffices to say that, for a given FER, we initially calculated the required $\frac{E_b}{N_0}$ values when we have perfect carrier phase tracking from Equation (2). We then calculate the required $\frac{E_b}{N_0}$ when there is imperfect carrier phase tracking. The difference between the two is the radio loss. Table 2 shows the required $\frac{E_b}{N_0}$ with perfect phase tracking. Figure 2 shows the high rate radio losses for the different codes as a function of LSNR. As we can see from this figure, the high rate radio losses for the codes with the same block length are the same for a given LSNR.

Code	Required $\frac{E_b}{N_0}$ (dB)
(8920,1/3), FER=3×10^{-4}	0.43
(8920,1/6), FER=3×10^{-4}	-0.11
(1784,1/3), FER=6×10^{-5}	0.84
(1784,1/6), FER=6×10^{-5}	0.33

Table 2. Required $\frac{E_b}{N_0}$ for Different Turbo Codes, Perfect Carrier Phase Tracking

At low data rates the radio loss becomes independent of the data rate (see [2]). The low rate radio losses are shown in Figure 3.

IV. Simulation Results: Residual Carrier

The results of radio loss simulations for the residual carrier case are shown in Figures 4a and 4b. As we can see from these Figures for a given data rate, LSNR value and block size, the losses are the same. This indicates that the radio losses are a function of block size and not the code rate. This is not surprising because as Figures 2 and 3 indicated the high rate radio loss depends only on the block length of the code and the low rate radio loss is code independent. Therefore, we should expect that the radio losses for codes of same block length to be the same for a given data rate.

The similarity of the shape of radio loss curves as a function of data rate for different LSNR values led us to normalize these curves by their approximate span. The results of this normalization are shown in Figures 5a through 5d. These figures indicate that by knowing the span of the radio loss curves and data rate for the bandwidth of 10Hz, for each code we can calculate the radio loss for any given code. In addition, we have normalized the data rate to the bandwidth frame size product, that is, we divide the data rate by the product of bandwidth with the frame size. (For example if the block size is 1784 and the bandwidth is 10 Hz and the data rate is 89200 bits per second then the normalized rate, expressed in terms of frames per bandwidth, is 5.) This represents the frame rate to bandwidth ratio. We then plot the normalized radio losses against it (see Figure 6). As we can see from Figure 6 the shape of the normalized curves for both 1784 and 8920 codes are the same. Therefore, we may conclude that by knowing the bandwidth, the code block size, and the span of the radio loss curve (which depends on the LSNR) for any given data rate we can calculate the radio loss by using Figure 6.

The reason that the frame rate to bandwidth ratio is the correct measure of determining whether the loop operates in the low rate or the high rate regime is that this ratio represents the numbers of frame per independent value of tracking phase error. The loop bandwidth represents the rate of independent values of tracking phase error. Therefore, by dividing the frame rate by the bandwidth we obtain the number of frames per independent value of the tracking phase error. If this ratio is high (i.e., many frames per independent phase error) the system operates under the high rate regime. Conversely when the ratio is low the system operates under the low rate regime.

V. Simulation Results: Suppressed Carrier

The results for the suppressed carrier simulations are presented in Figure 7. Two things are immediately obvious from looking at these curves and comparing them with the curves for the residual carrier. First of all, the losses are higher than those for the residual carrier with 17dB loop SNR. Second of all the curves are not as smooth as those for the residual carrier, especially at high data rates. There is a common cause for these two events: the fact that Costas loops suffer half cycle slips. Since Costas loops estimate twice the phase of the carrier, there is always a 180 degrees phase ambiguity during tracking. Since the decoder that was used in these simulations did not have the capability to detect whether there was a 180 degree phase shift on the received symbols, it produced many errors. These errors were especially pronounced at high data rates where a single 180 degree phase shift affects many frames.

In practice there will be frame synchronization markers that will allow the decoder to resolve this phase ambiguity. However, while it is easy to perform the simulations in such a manner as to prevent occurrence of half cycle slips (by taking the absolute value of the cosine of the tracking phase error) it was decided to present the curves with the half cycle slips included. Since the curves in Figure 7 are supposed to represent upper bounds for radio losses for suppressed carrier modulation, and since the half cycle slips only add to the error then use of radio losses that include half cycle slips as upper bounds is still valid. Note that in practice at high data rates for suppressed carrier there is enough power in the data as to increase the Costas loop SNR much higher than the 17dB simulated here. For example, at 40kbps a rate 1/6 code with a required bit SNR of 0dB and a receiver loop bandwidth of 10Hz produces a loop SNR of 30dB.

As for the curves in Figure 7, they indicate that, even with the half cycle slips and the low LSNR of 17dB, the losses for the turbo codes are not very high (about 1.2dB to1.3dB at high data rates). This, along with the results in the previous section, indicates that the turbo codes are reasonable for deep space applications.

VI. Conclusions

In this paper we have presented the radio loss results obtained through simulation for (8920,1/3), (8920,1/6), (1784,1/3) and (1784,1/6) turbo codes for both residual carrier and suppressed carrier modulation tracking cases for different data rates. The results indicate that for residual carrier cases the radio losses could be modeled rather elegantly and simply as a function of the code and the data rate. The suppressed carrier results indicate that half cycle slips which could occur during suppressed carrier operations could adversely affect the performance of the codes if no way of detecting such slips (such as frame synchronizers) are incorporated into the decoder.

Acknowledgments

The authors would like to thank Dr. Todd Chauvin and Adina Matache for providing them with the basic turbo code simulation program. They would also like to thank Dr. Sam Dolinar his invaluable advice regarding the simulations.

References:

[1] C. Berrou, A. Galvieux, and P. Thitimajshima, "Near Shannon Limit Error Correcting Coding and Decoding: Turbo-codes," *IEEE International Conference on Communications,* pp 1064-1070, Geneva, Switzerland, May 1993.

[2] J. H. Yuen, *Deep Space Telecommunications System Engineering*, pp 268-270 Pelnum Press, New York, New York, 1983.

[3] P. Kinman, "Simulation of Carrier, Subcarrier and Symbol Loop Phase Errors," Internal Memorandum, Case Western Reserve University, Cleveland, OH, and Jet Propulsion Laboratory, Pasadena, California, April 2, 1998.

[4] S. Shambayati, "Effects of Oscillator Phase Noise on a Phase Locked Loop with a Second Order Loop Filter: Simulation Results," Jet Propulsion Laboratory, Interoffice Memorandum 3311-SS02-00, Pasadena, California, May 4, 2000.

Figure 1. Turbo Codes FER vs Eb/No, fit and simulation

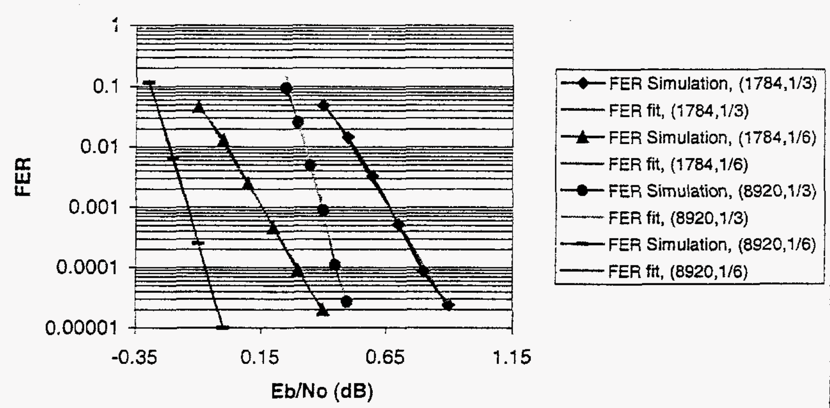
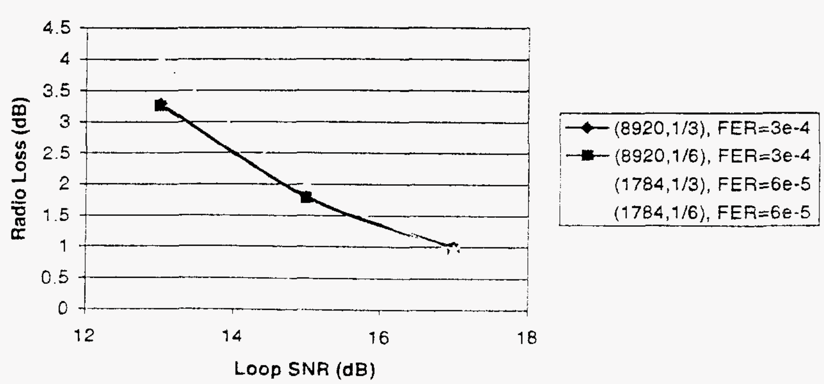


Figure 2. High Rate Radio Loss vs Loop SNR, Analysis



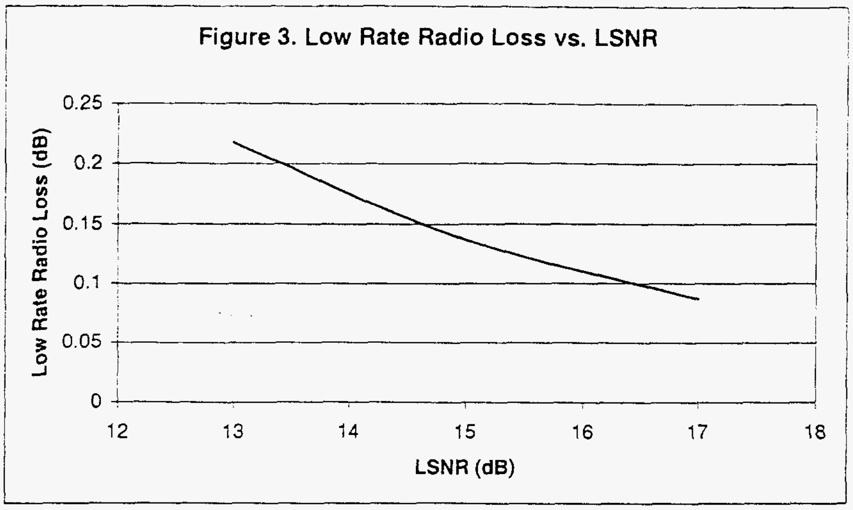


Figure 3. Low Rate Radio Loss vs. LSNR
Low Rate Radio Loss (dB)
0.25
0.2
0.15
0.1
0.05
0
12
13
14
15
16
17
18
LSNR (dB)

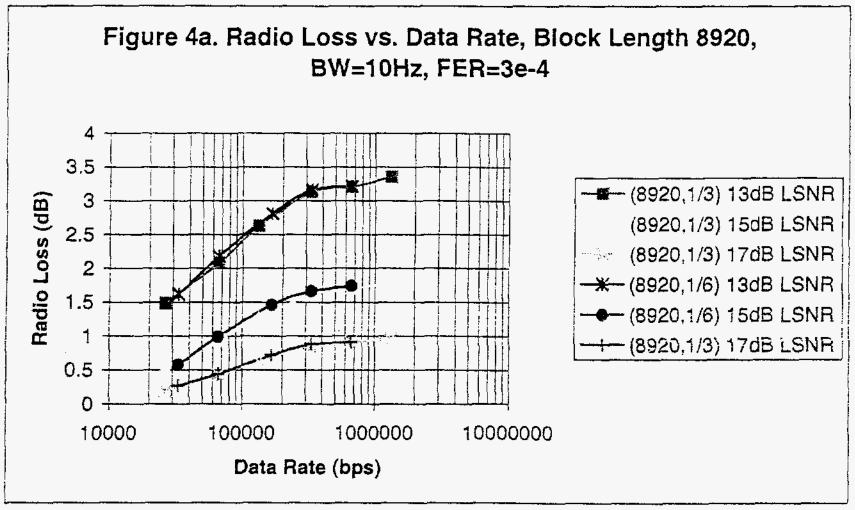


Figure 4a. Radio Loss vs. Data Rate, Block Length 8920, BW=10Hz, FER=3e-4
Radio Loss (dB)
4
3.5
3
2.5
2
1.5
1
0.5
0
10000
100000
1000000
10000000
Data Rate (bps)
(8920,1/3) 13dB LSNR
(8920,1/3) 15dB LSNR
(8920,1/3) 17dB LSNR
(8920,1/6) 13dB LSNR
(8920,1/6) 15dB LSNR
(8920,1/3) 17dB LSNR

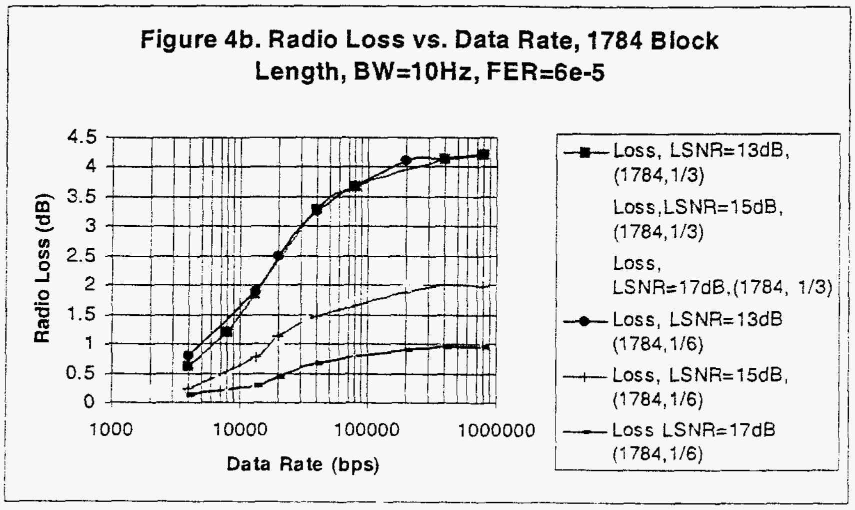


Figure 4b. Radio Loss vs. Data Rate, 1784 Block Length, BW=10Hz, FER=6e-5
Radio Loss (dB)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1000
10000
100000
1000000
Data Rate (bps)
Loss, LSNR=13dB, (1784,1/3)
Loss,LSNR=15dB, (1784,1/3)
Loss, LSNR=17dB,(1784, 1/3)
Loss, LSNR=13dB (1784,1/6)
Loss, LSNR=15dB, (1784,1/6)
Loss LSNR=17dB (1784,1/6)

Figure 5a. Normalized Loss vs. Data Rate, (8920,1/3), BW=10Hz, FER=3e-4

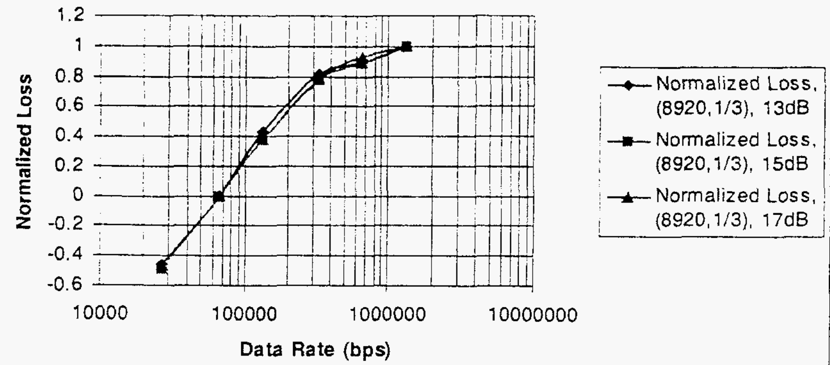


Figure 5b. Normalized Radio Loss vs Data Rate, (8920,1/6), BW=10Hz, FER=3e-4

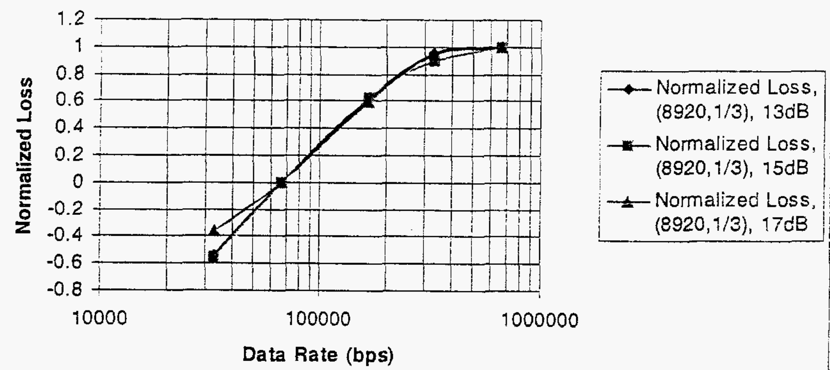


Figure 5c. Normalized Loss vs Data Rate, (1784,1/3) Code, BW=10Hz, FER=6e-5

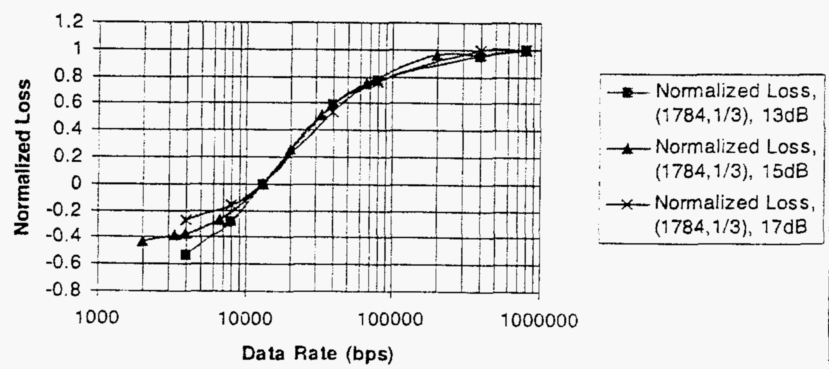


Figure 5d. Normalized Loss vs Data Rate, (1784,1/6), BW=10Hz, FER=6e-5

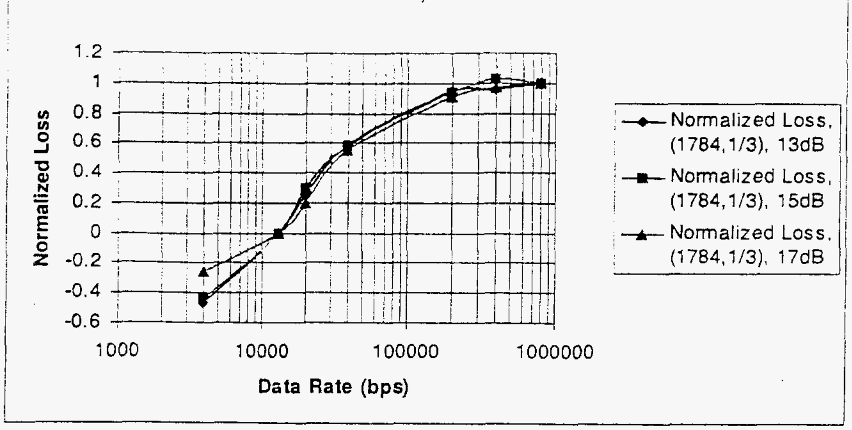


Figure 6. Normalized Loss vs Frame Rate/BW, (8920,1/3) and (1784,1/3), BW=10Hz

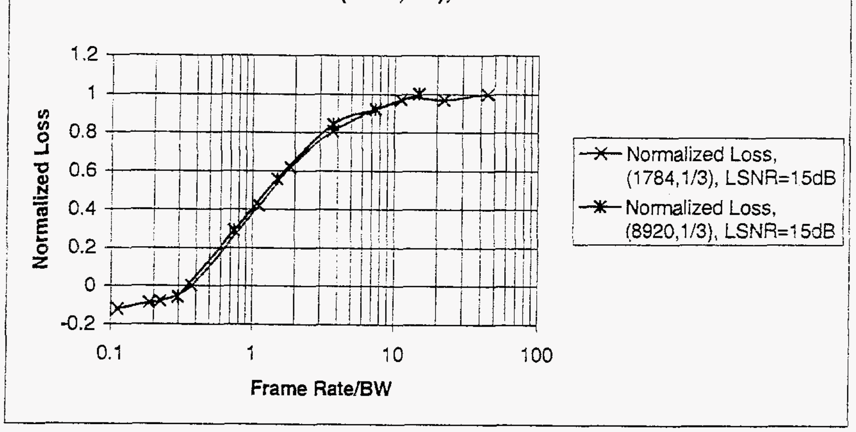


Figure 7. Radio Loss vs. Data Rate, Suppressed Carrier, BW=10Hz, LSNR=17dB